**Key words**: absolute geometry, Euclidean geometry, Euclid’s geometry, Cartesian plane over hyperreals, semi-Euclidean plane, axioms for geometry, Euclid, Hilbert, Hartshorne, Borsuk, Szmielew, Tarski, Thales’ theorem, area method, co-side theorem, automated theorem proving, proportion, real numbers, *Elements* Book VI, secondary school curriculum, prospective teachers of mathematics

**Abstract**

This thesis is dedicated to the basics of plane synthetic geometry and concerns congruent triangles, parallel lines, and similar figures. It originates from two basic observations. First, while synthetic geometry is a vital part of high school curricula, academic courses for prospective teachers provide mere information on the synthetic approach, enumerating axioms, preferably of the Hilbert system. Second, school textbooks misrepresent the foundations of geometry and hardly employ the benefits of IT technologies. With this thesis, we aim to revise the curriculum for high schools and propose a course on synthetic geometry for prospective mathematics teachers. Concerning the theory of similar figures, we go beyond the standard axiomatic approach and bring in a primer of automatic theorem proving.

In chapter 1, we spell out a course on synthetic geometry for prospective teachers based on Hilbert’s axioms in a modernized form developed by Robin Hartshorne. It contains results about congruent triangles, parallel lines, concurrent lines in a triangle, and Thales’ theorem for commensurable line segments. It includes an extensive overview of Book I of Euclid’s Elements, focused on basic concepts, topics related to the Pasch axiom, criteria for congruent triangles, algebra of line segments and angles, and the Fifth Postulate. We also introduce Borsuk and Szmielew’s and Tarski’s axioms of Euclidean geometry and compare systems of Euclid, Hilbert, Borsuk and Szmielew, and Tarski from the axiomatic perspective. Discussing the parallel postulate, we present a model of a non-Euclidean plane in which angles in a triangle add up to $π$; it is a subspace of the Cartesian plane over the non-Archimedean field of hyperreal numbers $R^{\*}$. Due to similar triangles, we extend the course with the automatic theorem proving. These topics relate to Thales’ theorem; we detail them through Chapters 3 and 4.

Our recommendations for the secondary school curriculum include using the application *euclidea*, the rule that criteria for congruent triangles precede the parallel postulate, and using graphic patterns related to Euclid's proposition VI.1 and the co-side theorem.

Thales' theorem is a turning point between Greek and modern mathematics. It embodies Euclid's method of proportion that modern geometry considers inconsistent. Therefore, the 20th-century systems of geometry employ the arithmetic of line segments or the continuity axiom to prove Thales' theorem. In Chapter 2, we study these proofs, showing they are quite involved and hardly match school textbooks.

In this thesis, we present an alternative solution: apply the area method that recovers Euclid's proportion with its simple arguments and adopts the highest standards of rigor. In Chapter 3, we present the area method in an axiomatic fashion and define a model of the theory; in Chapter 4, we include proofs of propositions from Book VI using the GCLC automated theorem-proving program.

Chapter 5 summarizes the lessons learned from previous chapters on education.