Line Arrangements in Algebraic Terms
Summary of the Thesis

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The present PhD thesis is devoted to line arrangements in the complex projective plane considered in the context of contemporary algebraic problems. By a line arrangement we understand a finite set of pairwise distinct lines in the plane with the intersection points of those lines. This subject can be considered as one of the most classical in geometry, for instance Pappus theorem tells us that there exists a specific symmetric arrangement of 9 lines and 9 points with peculiar combinatorial and geometrical properties. In the thesis we focus on three modern aspects related to line arrangements, namely the containment problem for symbolic powers of homogeneous ideal associated with singular points of line arrangements, parameter space of Böröczky's arrangements of lines (these objects are important in the context of extremal combinatorial problems in the plane, for instance the orchard problem), and the freeness of plane curves in Saito's sense. The main contribution of the thesis can be summed up as follows:

1) We describe the parameter spaces of Böröczky's arrangements of $d \in \{13, 14, , 16, 18, 24\}$ lines and we show that these arrangement cannot be represented over the rational numbers.

2) We prove that the radical ideals I associated with the triple intersection points of Böröczky's arrangement of $d \in \{6, ..., 11\}$ do not lead to new examples of non-containment $I^{(3)} \subset I^2$, where $I^{(3)}$ is the third symbolic power of I. Moreover, we show that the containment problem is not determined by the weak combinatorics of arrangements.

3) We provide a characterization of all those Böröczky arrangements of d lines which are free in the sense of Saito.

In the last chapter, which can be considered as experimental in its nature, we study supersolvable line arrangements and a natural question about the existence of a minimal complement of a given line arrangement to the supersolvability.